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Non-Gaussianity in the Cosmic Microwave Background Temperature Fluctuations from Cosmic (Super-)Strings

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We compute analytically the small-scale temperature fluctuations of the cosmic microwave background from cosmic (super-)strings and study the dependence on the string intercommuting probability P. We develop an analytical model which describes the evolution of a string network and calculate the numbers of string segments and kinks in a horizon volume. Then we derive the probability distribution function (pdf) which takes account of finite angular resolution of observation. The resultant pdf consists of a Gaussian part due to frequent scatterings by long string segments and a non-Gaussian tail due to close encounters with kinks. The dispersion of the Gaussian part is reasonably consistent with that obtained by numerical simulations by Fraisse et al.. On the other hand, the non-Gaussian tail contains two phenomenological parameters which are determined by comparison with the numerical results for $P = 1$. Extrapolating the pdf to the cases with $P < 1$, we predict that the non-Gaussian feature is suppressed for small P.

I. INTRODUCTION

The imprint of cosmic strings on the cosmic microwave background (CMB) has been widely studied. Although cosmic strings are excluded as a dominant source of the observed large-angular- scale anisotropies [1], they could still be observable at small scales [2, 3, 4] with new arcminute CMB experiments, such as the South Pole Telescope [5] or the Atacama Cosmology Telescope [6]. Because the structure of a string network is highly nonlinear, it would naturally induce a non-Gaussian feature in the CMB fluctuations. In particular, a moving straight string produces discontinuities in CMB temperature, called the Kaiser-Stebbins effect [7], and a temperature gradient map has been suggested as a means for detecting such an effect [8] (see also [9]). The non-Gaussian feature would also appear in the bispectrum [10], which has attracted much attention in the CMB community [11]. Fraisse et al. [3] found that the probability distribution function (pdf) of the temperature fluctuations has a non-Gaussian tail and negative skewness. These non-Gaussian features may help us distinguish cosmic string signals from other secondary effects and hence enhance their observability.

Recently, cosmic superstrings have attracted much attention in the context of inflation in string theory [12, 13]. Cosmic superstrings have properties different from conventional field-theoretic cosmic strings. One of the observationally interesting differences is concerning the intercommuting probability P. It can be significantly smaller than unity for superstrings while $P = 1$ is normally assumed for field-theoretic strings (but see [14]). This difference may be used to distinguish superstrings from field-theoretic strings observationally.

In this paper, we compute analytically the pdf of the small-scale CMB temperature fluctuations and study its dependence on P. At small scales where the primary fluctuations are damped, only the integrated Sachs-Wolfe (ISW) effect is relevant. Because the contribution from loops was shown to be insignificant [3], we focus on the ISW effect of long string segments and kinks. We first present the basic formulae for the temperature fluctuations induced by long string segments and kinks [15] (section II), and follow the evolution of the number densities of segments and kinks by combining and extending a velocity-dependent one-scale model [16, 17] and a kink model [18] (section III). Then, in section IV, we derive the pdf showing that the results in [3] can be interpreted with our simple model. Also the P dependence of the pdf is presented and the non-Gaussianity is predicted to be suppressed for small P. Finally we summarize our results in section V.

II. TEMPERATURE FLUCTUATIONS DUE TO COSMIC STRINGS

First we summarize the basic formulae for the CMB temperature fluctuations due to cosmic strings, following [15]. We denote the position of a cosmic string by $\vec{r}(t,\sigma)$ where t and σ are the time and position on the string worldsheet, respectively. The equations of motion and constraints in a flat spacetime are given by,

$$\ddot{\vec{r}} - \vec{r}\,'' = 0, \tag{1}$$

$$|\dot{\vec{r}}|^2 + |\vec{r}\,'|^2 = 1, \tag{2}$$

$$\dot{\vec{r}} \cdot \vec{r}\,' = 0, \tag{3}$$

where the dot and prime denote the derivatives with respect to t and σ, respectively. Photons obtain or lose their energies due to the gravitational field of cosmic strings. The temperature fluctuation in the direction $\hat{n}$, $\Delta \equiv \Delta T/T$, due to a straight segment with the length ξ is written as

$$\Delta_{\rm seg}(\hat{n}) = 8\frac{v}{\sqrt{1-v^2}}\alpha_{\rm seg} G\mu \arctan\frac{\xi}{\delta}, \tag{4}$$

where G is the Newton constant, μ is the string tension, δ is the impact parameter of a photon ray, $v \equiv |\dot{\vec{r}}|$ is the velocity of the segment and

$$\alpha_{\rm seg} = \hat{n} \cdot \left(\frac{\vec{r}\,'}{|\vec{r}\,'|} \times \frac{\dot{\vec{r}}}{|\dot{\vec{r}}|} \right) \tag{5}$$

is a factor which represents the configuration of the segment and the direction of the line of sight and can be positive or negative. In the limit that the impact parameter is much smaller than the segment length, $\delta \ll \xi$, Eq. (4) is reduced to the well known formula,

$$\Delta_{\rm seg}(\hat{n}) = 4\pi \frac{v}{\sqrt{1-v^2}} \alpha_{\rm seg} G\mu. \tag{6}$$

In fact, Eq. (4) can be well approximated by Eq. (6) for $\delta \lesssim \xi$ while it approaches zero for $\delta \gtrsim \xi$. Therefore a segment with the length ξ has an effective cross section $\sim \xi^2$.

On the other hand, a kink can be modeled as a non-smooth junction of two straight segments with different directions, $\vec{r}\,'$ [15]. Then the temperature fluctuation with the impact parameter δ is

$$\Delta_{\rm kink}(\hat{n}) = -4G\mu\alpha_{\rm kink} \log \frac{\delta}{L_{\rm kink}} \Theta(L_{\rm kink} - \delta), \tag{7}$$

where $L_{\rm kink}$ is a distance between kinks. The step function $\Theta(L_{\rm kink} - \delta)$ represents the effect that the fluctuation becomes negligible far from the kink, and $\alpha_{\rm kink}$ represents the kink configuration,

$$\alpha_{\rm kink} = \hat{n} \cdot \vec{p}, \quad \vec{p} = \left[\frac{\vec{r}\,'}{|\vec{r}\,'|^2} \right]_{\sigma_{\rm kink}-0}^{\sigma_{\rm kink}+0}, \tag{8}$$

where $\sigma_{\rm kink}$ is the position of the kink and $\vec{p}$ characterizes the change of the direction of the string at the kink and represents the kink amplitude.

III. ANALYTIC MODEL OF COSMIC STRING NETWORK

In this section, we develop an analytic model which describes the behavior of a cosmic string network. First, the average string length ξ and the rms velocity $v_{\rm rms}$ are calculated using a velocity-dependent one-scale model [16, 17]. Then, the number of kinks in a horizon volume is calculated with an approach similar to [18] (see also [20, 21, 22]).

In the velocity-dependent one-scale model, a string network is assumed to consist of straight string segments with the average length ξ and the rms velocity $v_{\rm rms}$. This scale ξ is also assumed to characterize the interstring distance, that is, $\rho_{\rm seg} = \mu/\xi^2$, where $\rho_{\rm seg}$ is the energy density of string segments, respectively. In terms of ξ, the number of segments in a horizon volume is expressed as

$$N_{\rm seg} = \frac{1}{\xi^3 H^3} = \gamma^3, \tag{9}$$

where H is the Hubble parameter and we defined $\gamma \equiv 1/(\xi H)$.

The evolution of the network of segments is determined by the cosmic expansion and the energy loss due to loop formation. A loop formation can occur through the intercommutation of two segments or the self-intercommutation of a single segment. The characteristic timescale for loop formation is $\sim \xi/P v_{\rm rms}$. For a universe with the scale factor $a(t) \propto t^\beta$, the evolution equations for γ and $v_{\rm rms}$ are given by [16, 17]

$$\frac{t}{\gamma}\frac{d\gamma}{dt} = 1 - \beta - \frac{1}{2}\beta\tilde{c}P v_{\rm rms}\gamma - \beta v_{\rm rms}^2, \tag{10}$$

$$\frac{dv_{\rm rms}}{dt} = (1 - v_{\rm rms}^2) H \left[k(v_{\rm rms})\gamma - 2v_{\rm rms} \right], \tag{11}$$

where $\tilde{c}$ is a constant which represents the efficiency of the loop formation and $k(v_{\rm rms}) \approx (2\sqrt{2}/\pi)(1 - 8v_{\rm rms}^6)/(1 + 8v_{\rm rms}^6)$ is the momentum parameter [16]. Hereafter we assume a matter-dominated universe and set $\beta = 2/3$.

It is known that a string network approaches a "scaling" regime where the characteristic scale grows with the horizon size [24]. This means that γ and $v_{\rm rms}$ are asymptotically constant in time. Here we assume that the scaling behavior is already realized by the recombination time. From (10) and (11), we obtain the scaling values of γ and $v_{\rm rms}$ neglecting their time derivatives. For small $\tilde{c}P$ they can be approximately given as,

$$v_{\rm rms}^2 \approx \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\pi\tilde{c}P}{3\sqrt{2}}}, \quad \gamma = \frac{2v_{\rm rms}}{k(v_{\rm rms})} \approx \sqrt{\frac{\pi\sqrt{2}}{3\tilde{c}P}}. \tag{12}$$

We see that small P, which means the inefficient loop formation, leads to large γ and hence large $N_{\rm seg}$. From Eq. (12), we have the dependence $\rho_{\rm seg} \propto P^{-1}$, which is consistent with the result of numerical simulations in [23] while [17] obtained a relatively weaker dependence on P. Actually there is no consensus on the dependence on P and we argue the effects of this ambiguity on the pdf later.

Next, we consider the kink number evolution. Our approach is based on the idea of [18] although the formulation is somewhat different. Small scale structure on strings including kinks is also considered in [19] in a different approach. Kinks are formed on string segments when they intercommute and, simultaneously, some of the existing kinks are removed through loop formation. Furthermore, kinks decay due to stretching by the cosmic expansion and the emission of gravitational waves. Here we neglect the decay due to the gravitational wave emission and focus on the decay due to cosmic expansion since it is the most efficient decay process at the matter-dominated stage [18].

According to [25], the kink amplitude, $p = |\vec{p}|$ (see Eq. (8)), decays with cosmic expansion as $p(t) = p_{\rm f}(t/t_{\rm f})^{-\epsilon}$, where $t_{\rm f}$ and $p_{\rm f}$ are the formation time and the amplitude

at the formation, respectively, and

$$\epsilon \equiv \frac{2(1-2v_{\rm rms}^2)}{3} \approx \frac{2}{3}\sqrt{\frac{\pi\tilde{c}P}{3\sqrt{2}}}. \tag{13}$$

We count the number of kinks with amplitude $p_{\rm min} \le p \le p_{\rm max}$ where $p_{\rm min}$ and $p_{\rm max}$ are free parameters. Later we show that we need only the ratio $p_{\rm max}/p_{\rm min}$ for our calculation and it will be determined by comparing our pdf for $P = 1$ with that obtained by the numerical simulations [3]. Even if a kink is formed with $p_{\rm f} > p_{\rm min}$, the cosmic expansion reduces the amplitude gradually and eventually it is no longer counted as a kink after a time determined by $p(t) = p_{\rm min}$. Therefore, the kink number in a comoving volume $V(t) \propto a^3(t)$ is given by the following integral of the formation rate $d\bar{N}_{\rm form}(t,p)/dtdp$,

$$\bar{N}_{\rm kink} = \int_{p_{\rm min}}^{p_{\rm max}} dp \int_{t_0(p)}^{t} dt\ \frac{d\bar{N}_{\rm form}(t,p)}{dtdp}, \tag{14}$$

where $t_0(p) = t(p/p_{\rm max})^{1/\epsilon}$, and a barred quantity is a value in the comoving volume $V(t)$.

The formation rate of kinks, which is assumed here to be independent of p, is proportional to the loop formation rate, $d\bar{N}_{\rm loop}/dt$. Because the loop formation rate determines the rate of the loss of the energy of string segments, we have [18]

$$\frac{d\bar{N}_{\rm form}(t)}{dtdp} = \frac{q}{p_{\rm max}}\frac{d\bar{N}_{\rm loop}(t)}{dt} = \frac{q\tilde{c}Pv_{\rm rms}}{p_{\rm max}\alpha\xi}\frac{V}{\xi^3}, \tag{15}$$

where q is a constant which represents the efficiency of the kink formation and α is the average loop length in units of ξ. Performing the integrations in (14), the kink number in a horizon volume $N_{\rm kink} = \bar{N}_{\rm kink}/(VH^3)$ is

$$N_{\rm kink} \approx \frac{2q\tilde{c}Pv_{\rm rms}\gamma^4\epsilon}{3\alpha}\left(\frac{p_{\rm max}}{p_{\rm min}}\right)^{1/\epsilon}, \tag{16}$$

where we have assumed $\epsilon \ll 1$. Because $N_{\rm kink}$ is independent of time, the kink number is also scaling. This means that the average distance between kinks, $L_{\rm kink}$, evolves in proportion to the horizon scale. In fact $L_{\rm kink}$ is given by

$$L_{\rm kink} \equiv \frac{N_{\rm seg}\xi}{N_{\rm kink}} = \frac{1}{KH}, \tag{17}$$

$$K \equiv \frac{N_{\rm kink}\gamma}{N_{\rm seg}} = \frac{N_{\rm kink}}{\gamma^2}, \tag{18}$$

where the normalized linear kink density, K, is constant in time. Thus we have expressed the numbers of string segments and kinks in a horizon volume as functions of P.

IV. PDF OF CMB FLUCTUATIONS

Based on the elementary processes presented in section II and the network evolution model in section III, we calculate the pdf of the CMB temperature fluctuations due to string segments and kinks.

A photon ray is scattered by segments many times through its way from the last scattering surface to an observer. Hence the temperature fluctuation would behave like a random walk and the pdf from segments would be approximated by the Gaussian distribution. If we treat a segment as a particle with the cross section ξ^2 as we discussed below Eq. (6), the optical depth is

$$\tau = \int_0^{z_{\rm rec}} N_{\rm seg}H^3\xi^2\frac{dz}{H(1+z)} = \gamma\log(1+z_{\rm rec}), \tag{19}$$

where $z_{\rm rec} \approx 1100$ is the redshift at the recombination. This is estimated as $\gamma\log(1+z_{\rm rec}) \approx 16$ for $P = 1$ and larger for smaller P. Although the temperature change at each scattering is different depending on the factors $\alpha_{\rm seg}$ and v, it would be a good approximation to estimate the dispersion of the pdf using their statistical averages. Therefore, remembering Eq. (6), the dispersion is evaluated as,

$$\begin{aligned}\sigma &= \Delta_{\rm seg}\frac{\sqrt{\tau}}{2} \\ &= 2\pi\frac{v}{\sqrt{1-v^2}}\alpha_{\rm seg}G\mu\sqrt{\gamma\log(1+z_{\rm rec})} \\ &\approx 2\pi\alpha_{\rm seg}\sqrt{\log(1+z_{\rm rec})}\left(\frac{\pi\sqrt{2}}{3\tilde{c}P}\right)^{1/4}G\mu, \end{aligned} \tag{20}$$

where we have set $v = v_{\rm rms}$ and substituted Eq. (12) in the third equality, and $\Delta_{\rm seg}$ and $\alpha_{\rm seg}$ should be understood as their statistical averages, $\sqrt{<\Delta_{\rm seg}^2>}$ and $\sqrt{<\alpha_{\rm seg}^2>}$, respectively. Here it should be noted that the PDF from string segments should be, strictly speaking, the binomial distribution. However, for the number of trials evaluated above ($\sim$ 16), the deviation of the binomial distribution from the Gaussian distribution is negligibly small.

Next, let us consider the contribution from kinks. The temperature fluctuation depends on the impact parameter as given by (7). Solving for δ as a function of Δ, we have,

$$\delta(\Delta) = L_{\rm kink}e^{-|\Delta|/2\Delta_0}, \tag{21}$$

$$\Delta_0 \equiv 2\alpha_{\rm kink}G\mu. \tag{22}$$

Therefore the differential cross section with the temperature fluctuation Δ can be written as

$$\frac{d\sigma_{\rm kink}}{d\Delta} = \left|\frac{d}{d\Delta}\delta^2(\Delta)\right| = \frac{L_{\rm kink}^2}{\Delta_0}e^{-|\Delta|/\Delta_0}, \tag{23}$$

where $\alpha_{\rm kink}$ should again be understood as its statistical average of the kink configuration and $\sigma_{\rm kink}$ should not be confused with the coordinate on a string in section II.

Then the pdf of temperature fluctuations due to kinks is

$$\frac{dP_{\rm kink}}{d\Delta} = \int_0^{z_{\rm rec}} N_{\rm kink}H^3 \frac{d\sigma_{\rm kink}}{d\Delta}\frac{dz}{H(1+z)} = \frac{\gamma^2}{K\Delta_0}e^{-|\Delta|/\Delta_0}\log(1+z_{\rm rec}). \tag{24}$$

The normalization factor can be evaluated as

$$A \equiv \frac{\gamma^2 \log(1+z_{\rm rec})}{K\Delta_0} \approx \frac{3\pi\alpha\log(1+z_{\rm rec})}{8q\alpha_{\rm kink}G\mu}\left(\frac{3\sqrt{2}}{\pi\tilde{c}P}\right)^{3/2}\left(\frac{p_{\rm max}}{p_{\rm min}}\right)^{-(3/2)\sqrt{3\sqrt{2}/\pi\tilde{c}P}} \tag{25}$$

where we have used Eqs. (12), (16) and (18) in the second equality. Thus we have a pdf of the form,

$$\frac{dP_{\rm tot}}{d\Delta} = \frac{dP_{\rm G}}{d\Delta} + \frac{dP_{\rm NG}}{d\Delta}, \tag{26}$$

$$\frac{dP_{\rm G}}{d\Delta} = \frac{1}{\sqrt{2\pi}\sigma}e^{-\Delta^2/2\sigma^2}, \tag{27}$$

$$\frac{dP_{\rm NG}}{d\Delta} = Ae^{-|\Delta|/\Delta_0}, \tag{28}$$

where σ, Δ_0 and A are given by Eqs. (20), (22) and (25), respectively. $dP_{\rm G}/d\Delta$ is the Gaussian part due to frequent scatterings by string segments, and $dP_{\rm NG}/d\Delta$ is the non-Gaussian tail due to rare scatterings by kinks. Here, because $dP_{\rm NG}/d\Delta \ll 1$ as we see just below, we have normalized $dP_{\rm G}/d\Delta$ as $\int_{-\infty}^{\infty} d\Delta\ dP_{\rm G}/d\Delta = 1$.

In the limit $P \to 1$, we have

$$\sigma \approx 14G\mu, \quad A \approx 10\alpha_{\rm kink}^{-1}\left(\frac{p_{\rm max}}{p_{\rm min}}\right)^{-5.1}(G\mu)^{-1}, \quad \Delta_0 = 2\alpha_{\rm kink}G\mu, \tag{29}$$

where we have set $q = 2$, $\tilde{c} = 0.23$ and $\alpha = 0.1$ as their standard values [26]. Here we used $\alpha_{\rm seg} = 1/\sqrt{2}$ for the statistical average assuming a random distribution of $\vec{r}\,'$ and $\dot{\vec{r}}$. Contrastingly, the statistical average of $\alpha_{\rm kink}$ cannot easily be obtained because kinks evolve in time and their distribution is nontrivial. This problem is closely related to the number count of kinks discussed in section III and we will postpone it to the future work.

On the other hand, the pdf from numerical simulations [3] can be also described as Eqs. (26), (27) and (28) with

$$\sigma_{\rm sim} \approx 12G\mu,\ A_{\rm sim} \approx 0.03(G\mu)^{-1},\ \Delta_{0,\rm sim} \approx 9G\mu. \tag{30}$$

First, we note that the dispersion of the Gaussian part is well reproduced without any adjustable parameters. This would imply that our interpretation of the Gaussian part as frequent scatterings by segments is reasonable. Next, to compare the non-Gaussian part of Eqs. (29) with (30), we must specify the values of $\alpha_{\rm kink}$ and $p_{\rm max}/p_{\rm min}$. To estimate these parameters from the first principles is, however, beyond the scope of the present paper. This will be discussed in a forthcoming paper [27]. Here we just treat them as phenomenological parameters and put $\alpha_{\rm kink} = 4.5$ and $p_{\rm max}/p_{\rm min} = 2.3$ to make (29) and (30) consistent.

Before we discuss the P dependence of the pdf, let us consider the effect of finite angular resolution of CMB observation. As we saw above, a ray has to pass nearby a kink to have a large temperature fluctuation but it may not be resolved if the angular resolution is finite. This effect can be taken into account by assuming that the impact parameter δ cannot be smaller than a certain value $\delta_{\rm min}(z,\theta)$ determined by the redshift of a kink and the angular resolution θ,

$$\delta_{\rm min}(z,\theta) = \theta d_{\rm A}(z) = 2\theta H^{-1}(z)(\sqrt{1+z}-1), \tag{31}$$

where $d_{\rm A}(z)$ is the angular diameter distance. The largest fluctuation which can be generated by a kink at z is, then,

$$\Delta_{\rm max}(z,\theta) = 2\Delta_0\log\left(\frac{L_{\rm kink}(z)}{\delta_{\rm min}(z,\theta)}\right) = -2\Delta_0\log\left[2\theta K\left(\sqrt{1+z}-1\right)\right]. \tag{32}$$

Solving this in terms of z, we obtain the largest redshift of kinks which contributes to a specific value of Δ,

$$z_{\rm max}(\Delta,\theta) = \min\left[\left(1+\frac{e^{-|\Delta|/2\Delta_0}}{2\theta K}\right)^2 - 1, z_{\rm rec}\right], \tag{33}$$

where this is bounded by $z_{\rm rec}$ because we are considering the ISW effect. Then the pdf modified by a finite angular resolution is obtained by changing the integration range of Eq. (24) from $[0, z_{\rm rec}]$ to $[0, z_{\rm max}(\Delta,\theta)]$,

$$\frac{dP_{\rm kink}^{\rm res}}{d\Delta} = \int_0^{z_{\rm max}(\Delta,\theta)} N_{\rm kink}H^3\frac{d\sigma_{\rm kink}}{d\Delta}\frac{dz}{H(1+z)} = \frac{2\gamma^2}{K\Delta_0}e^{-|\Delta|/\Delta_0} \times\left[\log\left(1+\frac{e^{-|\Delta|/2\Delta_0}}{2\theta K}\right)+\frac{1}{2}\frac{e^{-|\Delta|/2\Delta_0}}{e^{-|\Delta|/2\Delta_0}+2K\theta}\right], \tag{34}$$

where we have assumed $z_{\rm max}(\Delta,\theta) < z_{\rm rec}$. In the limit of an infinite resolution, this reduces to Eq. (24), and in the opposite limit, we have

$$\frac{dP_{\rm kink}}{d\Delta} \approx \frac{3\gamma^2}{2\theta K^2\Delta_0}e^{-3|\Delta|/2\Delta_0}. \tag{35}$$

The effect of a finite resolution is important for $|\Delta| > \Delta_1$ where Δ_1 is defined by $e^{-\Delta_1/2\Delta_0}/(2\theta K) = 1$, that is, $\Delta_1 = -2\Delta_0\log(2\theta K)$. Setting $\theta = 0.42'$ which was adopted in [3], Δ_1 is estimated as $\approx 68G\mu$ for $P = 1$ and larger for smaller P. Thus the non-Gaussian tail steepens slightly for large $|\Delta|$ but the pdf of [3] is still well reproduced.

In Fig. 1, the dependence of the pdf on the intercommuting probability P are shown with angular resolution

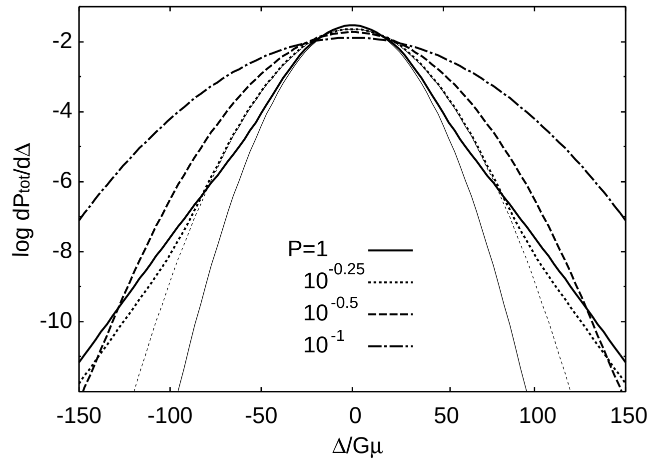

FIG. 1: Dependence of the pdf with angular resolution $\theta = 0.42'$ on the intercommuting probability P (thick lines). The respective Gaussian parts are plotted with thin lines for comparison. For $P = 1$ and $10^{-0.25}$, the pdfs deviate significantly from the Gaussian distribution. For $P \lesssim 10^{-0.5}$, pdfs are almost Gaussian.

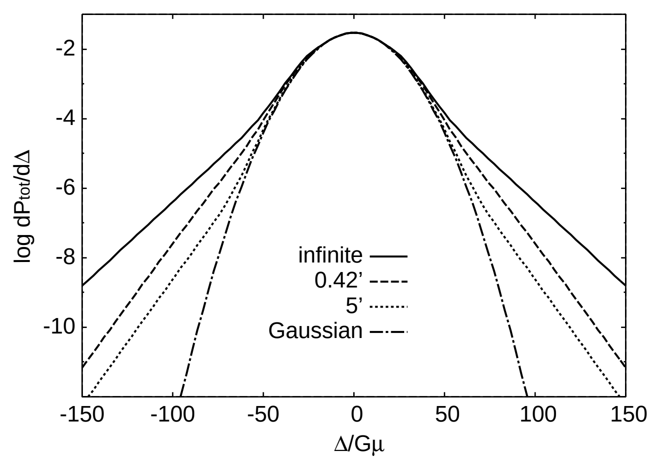

FIG. 2: Dependence of the pdf with $P = 1$ on angular resolution. The Gaussian part is also plotted for comparison.

$\theta = 0.42'$. The respective Gaussian parts are also plotted. As we see, as P decreases, the Gaussian dispersion increases and the contribution of the non-Gaussian tail is suppressed. For the case of $P = 1$, the pdf is almost Gaussian for $|\Delta| \lesssim 50G\mu$ while non-Gaussian tails can be seen for larger $|\Delta|$. Contrastingly, the pdfs are almost Gaussian for a wide range of $|\Delta|$ in the cases with $P \lesssim 10^{-0.5}$. Thus the non-Gaussianity could be a probe of the cosmic string property, P.

Fig. 2 shows the dependence on angular resolution with $P = 1$. We see that for a typical angular resolution, $5'$, of future observations such as *Planck* [28], non-Gaussian feature is highly suppressed even for $P = 1$ because kinks can not be resolved. Thus we would need observations with an arcminute resolution.

Note that our pdf is symmetric for positive and negative Δ and cannot reproduce the non-zero skewness reported in [3]. This is because we have assumed the long segments to be straight. In fact, for a straight segment, temperature fluctuation is symmetric between positive and negetive values as is seen from Eqs. (4) and (5). In this case, even if the number of scatterings is relatively small and the Gaussian approximation is not valid, skewness does not appear.

However, skewness would appear if we take the curvature of segments and its correlation with velocity into account [27, 29]. The reason is as follows.

First, if a segment has a curvature, the symmetry of temperature fluctuation between positive and negative values is broken. The asymmetry depends on the angle between the velocity and curvature vectors. Therefore the curvature cannot induce a skewness itself because the asymmetry would be canceled out by multiple scatterings of strings with various configuration of velocity and curvature vectors. However, if there is a correlation between velocity and curvature, there would be a nonzero expectation value for the asymmetry. This is the mechanism we believe the skewness is induced from.

To estimate the skewness, we need to extend the fluctuation formula Eq. (4) taking the curvature of a segment into account. With the nonzero expectation value of asymmetry, the deviation from the Gaussian distribution would lead to a nonzero skewness. The extention of Eq. (4) and the evaluation of the deviation from the Gaussian distribution due to the finite number of scatterings will be discussed in a separate article [27]. Nevertheless, it would be surprising that most of the features of the pdf obtained by the numerical simulations [3] can be interpreted by our simple model with just straight segments and kinks.

V. DISCUSSION AND SUMMARY

In this paper, we have computed analytically the pdf of small-scale CMB temperature fluctuations due to cosmic (super-)strings with a simple model with straight segments and kinks. Our purposes were to interpret the results of numerical simulations in [3] and study the effect of the string intercommuting probability P. We have combined and extended a velocity-dependent one-scale model and a kink model to calculate the numbers of string segments and kinks in a horizon volume consistently. Thus obtained pdf consists of a Gaussian component due to frequent scatterings by string segments and a non-Gaussian tail due to close encounters with kinks. The dispersion of the Gaussian part obtained by numerical simulations [3] is well reproduced without any adjustable parameters. On the other hand, the non-Gaussian tail contains two phenomenological parameters and we determined them by comparing it with that of the numerical result for $P = 1$ by Fraisse et al. [3]. Then we clarified the P dependence of the pdf and found that the non-Gaussian tail diminishes as P decreases.

Let us argue the ambiguity in our string network model in section III. The evolution of large- and small-scale structure has not been well understood either analytically or numerically. In particular, the dependence of γ, $v_{\rm rms}$ and K on P is quite important to derive the P dependence of the pdf by our formalism. As we pointed out below Eq. (12), the dependence of γ in our network model is consistent with that of [23] while [17] claims a relatively weaker dependence. However, they are consistent in that a small P results in large γ, $N_{\rm seg}$ and $\rho_{\rm seg}$. Then, from the second equation of Eq. (20), it would be robust that the dispersion of the Gaussian part increases as P decreases suppressing the non-Gaussian tail, assuming that $v_{\rm rms}$ would not differ significantly from $1/\sqrt{2}$ (see Eq. (12)). Anyway, the parameters of the pdf can be calculated by our formalism once γ, $v_{\rm rms}$ and K are given as functions of P. Thus it would be interesting to calculate the pdf using those functions obtained by other models and numerical simulations.

Our pdf is contributed only from the ISW effect of cosmic strings. Although the primary temperature fluctuations are substantially damped at small scales we consider here ($\sim O(1)$ min), other secondary fluctuations such the as Sunyaev-Zel'dovich effect would become important depending on the value of $G\mu$. To discuss further observational prospects of cosmic (super-)strings, it would be necessary to compare contributions from various secondary fluctuations.

Also it is important to compute other observational quantities with our simple formalism. In particular, the power spectrum contributed from cosmic strings has been calculated by many authors [2, 3, 4] and the comparison with them would further allow us to check the applicability of our formalism. As to non-Gaussianity, observationally more interesting quantities than the pdf would be higher-order correlation functions. The work along this direction is in progress [27].

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